Chapter 6

Polynomials Part B

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Author:	Shaun Carter
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This book is a work in progress.

6.1 Sums and Differences of Cubes, and Perfect Cubes

1 Distribute and fully simplify $(a+b)(a^2-ab+b^2)$ to prove the **sums of cubes** rule.

2 Distribute and fully simplify $(a-b)(a^2+ab+b^2)$ to prove the **differences of cubes** rule.

3 Distribute each of the following expressions.

a $(x-2)\left(x^2+2x+4\right)$
b $(x+6)\left(x^2-6x+36\right)$
c $(x-9)\left(x^2+9x+81\right)$
d $(2x+7)\left(4x^2-14x+49\right)$
e $(3x-10)\left(9x^2+30x+100\right)$
f $(5x-4)\left(25x^2+20x+16\right)$

4 Factor each of the following expressions.

a x^3+27
b x^3-64
c x^3+343
d $8x^3-125$
e $27x^3+512$
f $125x^3-1$

5 Distribute and fully simplify $(a+b)^3$ and $(a-b)^3$ to prove the **perfect cubes** rule.

6 Distribute each of the following expressions.

a $(x+3)^3$
b $(x-10)^3$
c $(x-7)^3$
d $(2x-4)^3$
e $(5x+1)^3$
f $(3x+5)^3$

7 Factor each of the following polynomials.

a $x^3-18x^2+108x-216$
b $x^3+15x^2+75x+125$
c $x^3-6x^2+12x-8$
d $27x^3-189x^2+441x-343$
e $125x^3+150x^2+60x+8$
f $8x^3+12x^2+6x+1$

8 Factor the following polynomials.

a $x^3+21x^2+147x+343$
b x^3+64
c x^3-125
d $x^3-27x^2+243x-729$
e x^3+22x^2+121x
f x^3-49x
g $18x^3-96x^2+128x$
h $40x^3-540x^2+2430x-3645$
i $32x^3+1372$
j $112x^3-847x$
k $1715x^3+3675x^2+2625x+625$
l $80x^3-270$

9 Factor the following polynomials.

a x^4-16
b x^4+10x^2+25
c x^6-2x^3+1
d x^6-64
e $x^6+15x^4+75x^2+125$
f x^9-512
g x^6+729
h $x^9-24x^6+192x^3-512$
i x^7-16x^4+64x
j $x^8+9x^6+27x^4+27x^2$
k x^9+216x^3
l $x^{11}-x^2$

6.2 Polynomial Division and the Remainder Theorem

1 Divide each polynomial by the given binomial. Use the remainder theorem to check your answer.

a $x^3 - 4x^2 - 2x + 3$ is divided by $(x - 4)$

b $x^3 + 7x^2 + 9x + 4$ is divided by $(x + 5)$

c $x^3 - 10x + 4$ is divided by $(x + 3)$

d $3x^3 - 18x^2 - 23x + 6$ is divided by $(x - 7)$

e $x^4 + 7x^3 - 14x^2 + 31x - 40$ is divided by $(x + 9)$

f $x^4 - 5x^3 + 11x - 7$ is divided by $(x + 1)$

g $5x^4 - 6x^3 - 5x^2 + 3x - 11$ is divided by $(x - 2)$

h $-6x^3 + 9x^2 + 11x - 18$ is divided by $(2x + 1)$

i $6x^3 - 13x^2 + 11x - 15$ is divided by $(3x - 5)$

j $10x^4 + 6x^3 - 9x^2 + 12x - 6$ is divided by $(5x - 2)$

2 Find a if the polynomial $x^3 - 8x^2 + ax + 9$ has a remainder of 6 when divided by $(x - 3)$.

3 $2x^3 + bx^2 + 9x + 3$ is divided by $(x + 8)$, resulting in a remainder of -5. Find b.

4 When $x^4 + cx^3 - 7x^2 + 3x + 16$ is divided by $(x - 6)$, the remainder is -2. Find c.

5 $x^4 - 2x^3 - 8x^2 + nx - 25$ has the same remainder when divided by $(x + 3)$ or $(x - 5)$. Find n.

6 $3x^4 + 15x^3 + 11x^2 + mx - 7$ has the same remainder when divided by $(x - 1)$ or $(x - 4)$. Find m.

7 Dividing $P(x) = x^3 + 3x^2 + px$ by $(x - 2)$ has a remainder of 4. Find the remainder when $P(x)$ is divided by $(x + 5)$.

Further Practice

- Create a division problem by choosing a polynomial $Q(x)$, a binomial $(x - a)$, and a number r. Then find $P(x)$ using the equation $P(x) = (x - a)Q(x) + r$.
- Give a partner $(x - a)$ and your result for $P(x)$, and have them find $Q(x)$ and r.

6.3 Factoring Polynomials and the Factor Theorem

1 Show that $(x-3)$ is a factor of $P(x) = x^3 + x^2 - 17x + 15$. Hence fully factor $P(x)$.

2 Show that $(x+2)$ is a factor of $P(x) = x^3 - 8x^2 + x + 42$. Hence solve the equation $P(x) = 0$.

3 Show that $(2x-3)$ is a factor of $P(x) = 2x^3 + 9x^2 - 2x - 24$. Hence fully factor $P(x)$.

4 Show that $(x+1)$ is a factor of $P(x) = x^4 + 7x^3 + 18x^2 + 20x + 8$. Hence solve the equation $P(x) = 0$.

5 Show that $(x-2)$ is a factor of $P(x) = x^4 - 2x^3 - 125x + 250$. Hence fully factor $P(x)$.

6 Show that $(2x+5)$ is a factor of $P(x) = 2x^4 - 19x^3 + 36x^2 + 112x - 320$. Hence fully factor $P(x)$.

7 Show that $(x+7)$ is a factor of $P(x) = x^5 + 7x^4 - 81x - 567$. Hence fully factor $P(x)$.

8 Factor the following polynomials.

a $x^3 - x^2 - 10x - 8$

b $x^3 - 12x^2 + 41x - 42$

c $x^3 - 6x - 9$

d $x^3 + 2x^2 - 21x + 18$

9 Solve the following equations.

a $x^3 - x^2 - 46x - 80 = 0$

b $x^3 - 12x^2 + 52x - 80 = 0$

c $2x^3 + 5x^2 - 11x - 14 = 0$

d $3x^3 - 2x^2 - 87x + 198 = 0$

10 Factor the following polynomials.

a $x^4 - 5x^3 - 9x^2 + 45x$

b $2x^4 + 16x^3 - 26x^2 - 280x$

c $x^4 + 9x^3 + 9x^2 - 41x - 42$

d $x^4 + 3x^3 - 31x^2 - 63x + 90$

11 Solve the following equations.

a $4x^4 - 36x^3 + 108x^2 - 108x = 0$

b $x^4 + 2x^3 - 16x^2 - 2x + 15 = 0$

c $5x^4 - 22x^3 + 13x^2 + 28x - 12 = 0$

d $7x^4 - 52x^3 - 259x^2 + 820x - 300 = 0$

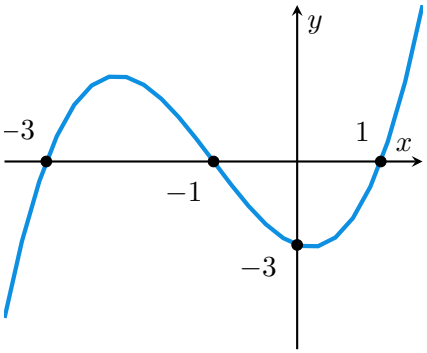
Further Practice

Create your own problems by distributing three or more binomial expressions. Then trade them with a partner.

6.4 Graphs of Polynomials

1 Determine the function for each of the following. Use technology to check your answer.

a



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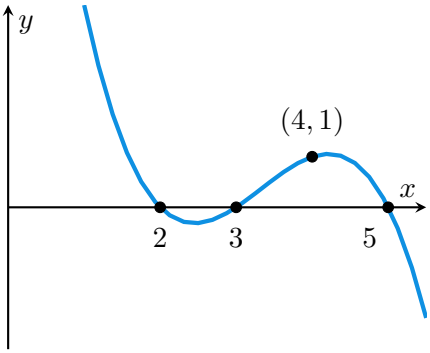

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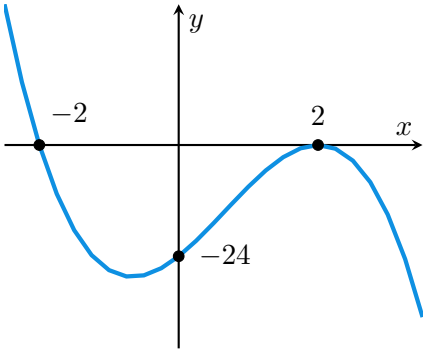

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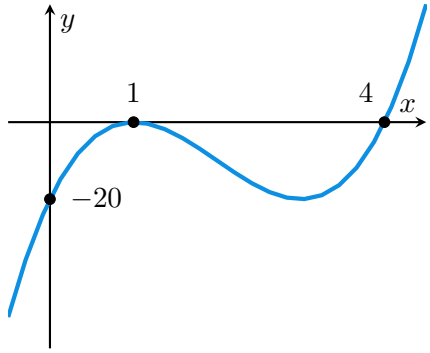

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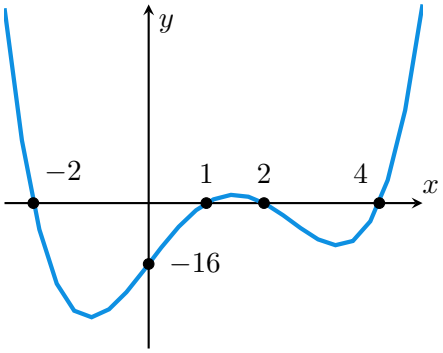

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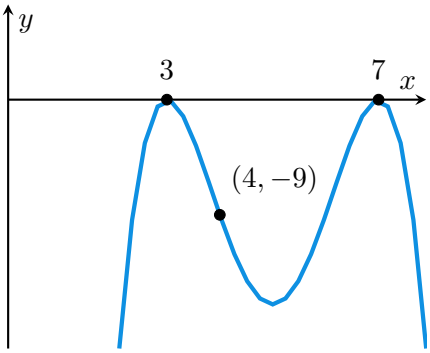

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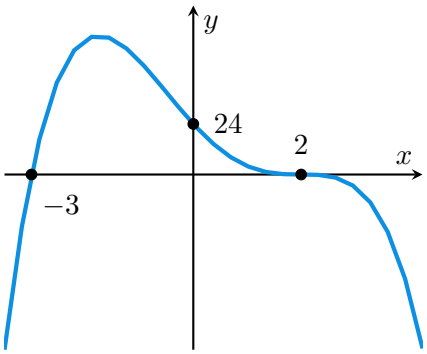
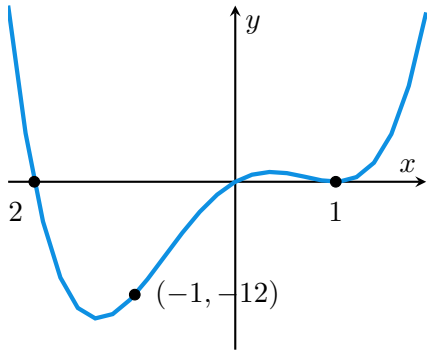


h



2 For each graph in question 1, find

i the domain for which the function is strictly positive, and

ii the domain for which the function is strictly negative.

3 Use technology to identify the local minima and maxima for each of the graphs in question 1.

4 For each graph in question 1, find

i the domain for which the function is strictly increasing, and

ii the domain for which the function is strictly decreasing.

5 For each of the following functions,

i use technology to help you sketch a graph,

ii label all intercepts,

iii label all endpoints and turning points, and

iv state the range of the function.

a $f : [5, 11] \to \mathbb{R}$, where $f(x) = (x - 6)(x - 8)(x - 11)$.

b $g : (0, 4) \to \mathbb{R}$, where $g(x) = x^3 - 5x^2 + 4x + 2$.

c $h : \{x : x \leq 5\} \to \mathbb{R}$, where $h(x) = x^4 - 10x^3 + 32x^2 - 36x + 9$.

6 Sketch a graph of each of the following functions without using graphing technology, showing all intercepts.

a $f(x) = (x + 2)(x - 5)(x - 4)$

b $f(x) = -x(x - 1)(x + 7)$

c $f(x) = 2(x + 5)(x + 1)^2$

d $f(x) = -3(x - 4)(x - 2)(x - 1)(x + 1)$

e $f(x) = (x - 1)^2(x - 5)^2$

f $f(x) = (x - 2)^3(x + 1)$

7 Use the factored form of the following functions to assist in graphing them.

a $f(x) = 2x^3 - 4x^2 - 10x + 12$

b $g(x) = -x^3 + x^2 + 22x - 40$

c $h(x) = x^4 - 3x^3 - 3x^2 + 7x + 6$

6.5 Imaginary Numbers

1 Evaluate the following expressions.

a i^{11} **b** i^{14} **c** $(-i)^{16}$ **d** $(-i)^{29}$

e i^{347} **f** i^{109} **g** $(-i)^{276}$ **h** $(-i)^{2837}$

i i^{-13} **j** $(-i)^{-27}$ **k** $(-i)^{-32}$ **l** i^{-75}

m $i^{23} \cdot i^{18}$ **n** $-i^{46} \cdot i^{28}$ **o** $(i^{67})^{395}$ **p** $(-i)^{57} \cdot (-i)^{68}$

2 Find the value of n that meets the given conditions.

a $i^n = i$, and $11 \leq n < 15$ **b** $i^n = -1$, and $24 \leq n < 28$

c $i^n = 1$, and $33 \leq n \leq 36$ **d** $i^n = -i$, and $45 \leq n \leq 48$

e $i^n = i$, and $29 < n < 34$ **f** $i^n = -1$, and $72 < n < 77$

g $i^n = 1$, and $-8 \leq n < -4$ **h** $i^n = -i$, and $-17 \leq n < -13$

3 Find all the values of n that meet the given conditions.

a $i^n = i$, and $25 \leq n < 35$ **b** $i^n = -1$, and $17 \leq n < 27$

c $i^n = 1$, and $12 \leq n \leq 24$ **d** $i^n = -i$, and $n \geq 104$

e $i^n = i$, and $-8 < n < 8$ **f** $i^n = -1$, and $-12 < n < 2$

g $i^n = 1$, and $-19 \leq n < -16$ **h** $i^n = -i$, and $n \leq -6$

4 Express the following using simplified exact values.

a $\sqrt{-25}$ **b** $\sqrt{-49}$ **c** $\sqrt{-64}$

d $\sqrt{-6}$ **e** $\sqrt{-2}$ **f** $\sqrt{-14}$

g $\sqrt{-75}$ **h** $\sqrt{-99}$ **i** $\sqrt{-20}$

j $\sqrt{-72}$ **k** $\sqrt{-250}$ **l** $\sqrt{-700}$

5 Solve the following equations.

a $x^2 + 9 = 0$ **b** $x^2 + 81 = 0$ **c** $x^2 + 16 = 0$

d $x^2 + 12 = 0$ **e** $x^2 + 18 = 0$ **f** $x^2 + 45 = 0$

g $3x^2 + 81 = 0$ **h** $-2x^2 - 686 = 0$ **i** $-8x^2 - 384 = 0$

6.6 Working with Complex Numbers

For the questions below, using the following values:
$z_1 = 5 + 3i$ $z_2 = 4 - 2i$ $z_3 = 3i - 2$ $z_4 = -7 - i$ $z_5 = 8i + 1$

1 Find $\text{Re}(z)$ and $\text{Im}(z)$ for $z = z_1, \ldots, z_5$

2 Add or subtract the following.

a	$z_1 + z_2$	**b**	$z_3 + z_4$	**c**	$z_5 - z_3$
d	$z_2 - z_5$	**e**	$z_1 + z_4$	**f**	$z_5 - z_1$
g	$2z_1$	**h**	$3z_4$	**i**	$3z_2 + 2z_5$
j	$7z_3 - z_2$	**k**	$z_4 + z_3 - z_1$	**l**	$z_1 + z_2 - z_3 + z_4 - z_5$

3 Multiply the following.

a	$z_1 \cdot z_2$	**b**	$z_3 \cdot z_5$	**c**	${z_4}^2$
d	$z_1 \cdot i$	**e**	$z_5 \cdot 2i$	**f**	$z_2 \cdot (2 + 3i)$
g	$z_3 \cdot z_4 \cdot z_5$	**h**	$z_1 \cdot (z_2 + z_3)$	**i**	$(z_4 - z_2) \cdot (z_3 - z_1)$
j	$(z_5 - z_2)^2 \cdot z_3$	**k**	$(z_1 + z_5) \cdot {z_3}^2$	**l**	$z_1 + {z_2}^2 + {z_3}^3$

4 Find $\overline{z}$, the complex conjugate, for $z = z_1, \ldots, z_5$.

5 Find $z \cdot \overline{z}$ for $z = z_1, \ldots, z_5$.

6 What do you notice happens when a complex number is multiplied by its conjugate?

Further Practice

Create your own complex numbers in the form $a + bi$, including a mix of positive and negative values. Perform the same operations on them as the questions in this exercise. Use technology to check your answers.

6.7 Dividing Complex Numbers

1 Write the following in the form $a+bi$.

a $\dfrac{1}{2+4i}$ **b** $\dfrac{1}{3-2i}$ **c** $\dfrac{1}{-7+5i}$

d $\dfrac{1}{-1+8i}$ **e** $\dfrac{1}{6-i}$ **f** $\dfrac{1}{-10-3i}$

2 Write the following in the form $a+bi$.

a $\dfrac{2}{3+7i}$ **b** $\dfrac{3i}{4-5i}$ **c** $\dfrac{2+3i}{-6+8i}$

d $\dfrac{4-2i}{-2+9i}$ **e** $\dfrac{-3+2i}{5-i}$ **f** $\dfrac{i-6}{-12+5i}$

g $\dfrac{3i-7}{3+7i}$ **h** $\dfrac{2+i}{-4+11i}$ **i** $\dfrac{5-2i}{8-3i}$

j $\dfrac{2-3i}{7i-5}$ **k** $\dfrac{9i-4}{3i+7}$ **l** $\dfrac{9+i}{8+2i}$

Further Practice

Create your own complex numbers in the form $a+bi$, including a mix of positive and negative values. Perform the same operations on them as the questions in this exercise. Use technology to check your answers.

3 Let $z=a+bi$.

a Find $\overline{z}$ in terms of a and b.

b Find $z\cdot\overline{z}$ in terms of a and b.

c Let $\dfrac{1}{z}=c+di$. Find c and d in terms of a and b.

4 Let $z_1=a_1+b_1i$ and $z_2=a_2+b_2i$.

a Show that $\overline{z_1}+\overline{z_2}=\overline{z_1+z_2}$.

b Show that $\overline{z_1}-\overline{z_2}=\overline{z_1-z_2}$.

c Show that $\overline{z_1}\cdot\overline{z_2}=\overline{z_1\cdot z_2}$.

d Show that $\dfrac{\overline{z_1}}{\overline{z_2}}=\overline{\left(\dfrac{z_1}{z_2}\right)}$.

6.8 Quadratic Equations with Non-Real Solutions

1 Solve the following equations.

a $(x-4)^2+9=0$ **b** $(x+7)^2+16=0$ **c** $-(x+1)^2-64=0$

d $(x+6)^2+12=0$ **e** $-(x-5)^2-18=0$ **f** $-(x+2)^2-48=0$

2 Solve the following equations by completing the square.

a $x^2-4x+13=0$ **b** $x^2+10x+29=0$ **c** $2x^2+16x+82=0$

d $x^2+6x+36=0$ **e** $10x^2+140x+690=0$ **f** $2x^2+20x+140=0$

g $x^2+19x+93=0$ **h** $2x^2-34x+156=0$ **i** $4x^2+20x+28=0$

3 Solve the following equations using the quadratic formula.

a $2x^2+3x+7=0$ **b** $3x^2-5x+13=0$

c $4x^2-12x+8=2x-6$ **d** $-2x^2+5x-4=0$

e $-5x^2-11x+8=-3x+15$ **f** $-5x^2-11x+8=x+2$

4 For each equation,

i determine the number and nature of the solutions, and

ii solve the equation.

a $x^2-10x+17=0$ **b** $2x^2-7x+8=0$ **c** $3x^2-18x+22=0$

d $x^2-12x+21=0$ **e** $-3x^2+2x-4=0$ **f** $-2x^2+6x-3=0$

g $x^2+3x+4=0$ **h** $-9x^2+30x-25=0$ **i** $4x^2+12x+9=0$

5 Find an equation with each of the following solutions.

a $-3\pm 2i$ **b** $4\pm 6i$ **c** $-7\pm 11i$

d $5\pm 2\sqrt{3}i$ **e** $12\pm 5\sqrt{2}i$ **f** $-3\pm 3\sqrt{5}i$

6 Solve the following equations.

a $x^3-5x^2+17x-13=0$ **b** $x^3+8x^2+9x-58=0$

c $x^3+3x^2+15x+13=0$ **d** $x^3-3x^2+36x+162=0$

e $x^4-11x^3+49x^2-21x-82=0$ **f** $x^4+13x^2-198x+328=0$

Further Practice

Now that you can find complex solutions, you can solve any quadratic equation. Choose real numbers a, b and c, and solve the equation $ax^2+bx+c=0$.